Title: Angle-Arc Relationships in the Circle

Brief Overview:

Students will use *The Geometer's Sketchpad* and the instructional activity sheets to explore the relationships between the measures of angles related to the circle and their intercepted arcs. This knowledge will be used to form hypotheses about the relationships and develop the formulas that describe each of the relationships.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to solve mathematical problems through the use of generalization from data to formulas.
- **Communications** Students will demonstrate the ability to express observed relationships in writing.
- **Reasoning** In Part I, students will follow logical mathematical reasoning which will enable them to determine the relationships between the measures of angles and their intercepted arcs. In Part II, students will then incorporate knowledge gained from Part I to explain why observed relationships are true.
- **Number Relationships** Students will recognize relationships between and among measurements derived from sketches formed by *The Geometer's Sketchpad*. These relationships will form the basis for the hypotheses and formulas which they will generate.
- **Algebra** Students will develop formulas to represent verbal relationships.

Grade/Level:

Grades 9 - 10, Geometry

Duration/Length:

This lesson will take three class periods of approximately 45 minutes each.

Prerequisite Knowledge:

Students should have working knowledge of *The Geometer's Sketchpad* and the terminology included on the introductory page of the instructional sheet packet.

Objectives:

Students will:

- work cooperatively in pairs.
- collect and record data using existing animated sketches.
- construct sketches and save them on disk and/or in print.
- formulate hypotheses bases on their collected and observed data.
- complete all activities in the instructional activity packet.

Materials/Resources/Printed Materials:

- Computers with *The Geometer's Sketchpad*
- Four animated sketches for use with *GSP*
- Ability to save sketches on disk and/or print sketches
- Student Instructional Activity Packet consisting of an introduction and seven investigation.
- Instructions to the teacher

Development/Procedures:

- Prior to the initial class, the four animated sketches must be accessible to the students.

- Keeping in mind that this activity is designed for a maximum of three class periods, students should proceed at a pace that will enable completion of the task within this timeframe.

- The teacher should collect the following: the completed instructional activity packets, one per student; sketches for investigations 5 through 7 printed and/or on disks, one per group.

Evaluation:

Teachers are encouraged to consider the student-created sketches for investigations 5 through 7 as alternative assessments. Students are encouraged to include a copy of their created sketches in their portfolios. Typical paper-and-pencil assessment can follow the activity.

Extension/Follow Up:

- Especially in honors level classes, students should be encouraged to develop proofs for their hypotheses.

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Angle-Arc Relationships in the Circle

Introduction To The Teacher

In designing this activity, our objective was to create a package which the teacher could incorporate into teaching required material for her/his class. It is our hope that the teacher will be free to function as a facilitator.

The package consists of four animated sketches for *The Geometer's Sketchpad* and a student instructional activity packet. The packet includes an introduction and seven investigations. As you will see, students are requested to save copies of their own sketches for the last three investigations. This can be done in whatever manner is convenient for you.

Angle-Arc Relationships in the Circle
Introduction

The relationships between angles and arcs of a circle will be studied through seven investigations using ***The Geometer's Sketchpad***. In Part I, which consists of the first four investigations, you are provided with sketches which you will manipulate to collect data and form hypotheses. In Part II, which consists of the last three investigations, you will construct your own sketches in order to form hypotheses.
The following terms should be familiar to you:

CIRCLE:	a set of points in a plane equidistant from a fixed point
CENTER OF A CIRCLE:	the fixed point from which the points of a circle are equidistant
RADIUS:	a segment with one endpoint at the center of the circle and one endpoint on the circle
CHORD:	a segment whose endpoints are on the circle
DIAMETER:	a chord containing the center of the circle
SECANT LINE:	a line intersecting a circle at two points
TANGENT LINE:	a line in the plane of the circle intersecting the circle at one point
SEMICIRCLE:	an arc of measure 180 degrees
MINOR ARC:	an arc of measure less than 180 degrees
MAJOR ARC:	an arc of measure greater than 180 degrees

You should also have some familiarity with the basic workings of ***The Geometer's Sketchpad***. You need to be able to do the following:

CONSTRUCT
- **segments**
- **points on objects**

MEASURE
- **angles**
- **arc angles**
- **calculate**
- **tabulate**

Angle-Arc Relationships in the Circle

Investigation #1

The first type of angle to investigate is called a **CENTRAL ANGLE**. A central angle is one whose vertex is at the center of the circle. Each of its rays intersects the circle at one point. The arc inside the angle is called the **INTERCEPTED ARC**.

To discover the relationship between the measure of a central angle and its intercepted arc, load ***CENTRAL ANGLE*** from your disk or the main menu. At the bottom of the sketch is a table with one set of values entered. Animate the demonstration. Periodically halt the animation so that you can expand the table with the displayed values. Double-click on the table to expand the table with the displayed values. When you feel you have enough information on which to form a hypothesis about the relationship between the central angle and its intercepted arc, complete the remainder of this worksheet.

The center of the circle is point _____.

The vertex of the angle is point _____.

The rays of the angle intersect the circle at points _____ and _____.

The intercepted arc is arc _____.

Copy the values from the Sketchpad table to your table below:

Measure of Arc DC						
Measure of Angle DAC						

In your own words, describe the relationship between the measure of a central angle and its intercepted arc.

Angle-Arc Relationships in the Circle
Investigation #2

The second type of angle to investigate is called an **INSCRIBED ANGLE**. The vertex of an inscribed angle is a point on the circle, and its rays either intersect the circle in one other point or are tangent rays. A tangent ray must lie on a tangent line. As with the central angle, the arc inside the angle is called the intercepted arc. You will notice that the inscribed arc is at times a major arc. Since all major arcs **MUST** be named by three points and all other arcs **CAN** be named by three points, the intercepted arc is always named by three points.

To discover the relationship between the measure of an inscribed angle and its intercepted arc, load ***INSCRIBED ANGLE*** from your disk or the main menu. At the bottom of the sketch is a table with one set of values entered. Animate the demonstration. Periodically halt the animation so that you can expand the table with the displayed values.

WARNING!!! Because of the roundoff feature of the measures given, the relationship may not seem exact at times.

When you feel you have enough information on which to form a hypothesis about the relationship between the inscribed angle and its intercepted arc, complete the remainder of this worksheet.

The center of the circle is point ___. The vertex of the angle is point ___.

The intercepted arc is arc _____.

Copy the values from the Sketchpad table to your table below:

Measure of Arc PEN						
Measure of Angle PDN						

In your own words, describe the relationship between the measure of an inscribed angle and its intercepted arc.

Using an appropriate choice of variables, express this relationship as a formula. Be sure to define your variables.

Angle-Arc Relationships in the Circle
Investigation #3

The third type of angle to investigate is called an **INSIDE ANGLE**. The vertex of an inside angle is a point in the interior of a circle. One type of inside angle is the central angle, but that is a special case. Before continuing this discussion, load ***INSIDE ANGLE*** from your disk or the main menu.

Notice that arc DHE, which is the intercepted arc for angle EQD is not congruent to arc CGF, which is the intercepted arc for angle CQF. However, angles EQD and CQF are congruent! Complete the following:

Angles EQD and CQF are congruent because they are _____ angles.

Therefore, the measure of an inside angle must be related not only to its intercepted arc but also that of its vertical angle. This relationship is not as obvious as the two previous ones. As a hint, the relationship involves two arithmetic operations. Animate the sketch and expand the table as before.

WARNING!!! Because of the roundoff feature of the measures given, the relationship may not seem exact at times.

When you feel you have enough information on which to form a hypothesis about the relationship between the inside angle and its intercepted arcs, complete the remainder of this worksheet.

The common vertex of the angles is point _____.

The name of one angle is angle _____.

The intercepted arc of this angle is arc _____.

The name of the vertical angle is angle _____.

The intercepted arc of the vertical angle is arc _____.

Copy the values from the Sketchpad table to your table below.

Measure of Arc CGF						
Measure of Arc DHE						
Measure of Angle CQF						
Measure of Angle EQD						

In your own words, describe the relationship between the measure of an inside angle and the measures of its intercepted arcs.

Using an appropriate choice of variables, express this relationship as a formula. Be sure to define your variables.

Angle-Arc Relationships in the Circle
Investigation #4

The last type of angle to investigate is called an **OUTSIDE ANGLE**. The vertex of an outside angle is a point in the exterior of a circle, and its rays are either secant or tangent rays. Before continuing this discussion, load ***OUTSIDE ANGLE*** from your disk or the main menu.

Notice that there are two intercepted arcs for angle RCT. Therefore, the measure of an outside angle must be related to the measures of both arcs. As a hint, the relationship again involves two arithmetic operations. Animate the sketch and expand the table as before.

WARNING!!! Because of the roundoff feature of the measures given, the relationship may not seem exact at times.

When you feel you have enough information on which to form a hypothesis about the relationship between the outside angle and its intercepted arcs, complete the remainder of this worksheet.

The vertex of the angle is point _____.

The intercepted arcs are arc _____ and arc _____.

Copy the values from the Sketchpad table to your table below.

Measure of Arc TVR						
Measure of Arc US						
Measure of Angle RCT						

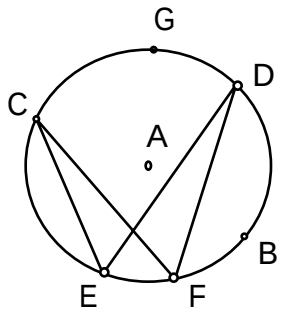
In your own words, describe the relationship between the measure of an outside angle and the measures of its intercepted arcs.

Using an appropriate choice of variables, express this relationship as a formula. Be sure to define your variables.

Angle-Arc Relationships in the Circle
Part II
Investigation #5

In your previous investigations into the relationships between angles and arcs of the circle, you were able to access sketches which had been created for you. In this second part you must create your own sketches in order to do the investigation. Be sure to save and/or print your sketches as directed by your teacher.

Create a sketch like the one below:



Measure angle CED, angle CFD and arc CGD.

Either through animation or by manually moving point C or D about the circle, observe the changes in your measures. When you feel you have enough information on which to form a hypothesis about the angles in the diagram, complete the remainder of this worksheet.

Be sure to save and/or print your sketches as directed by your teacher.

In Part I you investigated four types of angles: central, inscribed, inside, and outside. Which of these types of angles are angles CED and CFD?

How is the measure of this type of angle related to its arc?

To what arc is the measure of angle CED related?

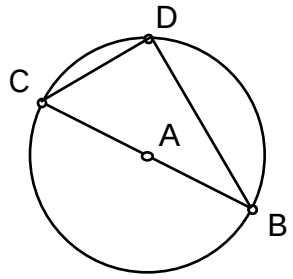
To what arc is the measure of angle CFD related?

In your own words, explain why the relationship between the measures of angles CED and CFD is true.

Angle-Arc Relationships in the Circle
Part II
Investigation #6

The arc inside an angle is called the intercepted arc. An angle is **INSCRIBED IN AN ARC** when the endpoints of the arc are points on the sides (rays) of the angle and the vertex of the angle is another point on the arc.

Create a sketch like the one below. Segment CB must be a diameter. Be sure to save and/or print your sketch as directed by your teacher.



Angle CDB is inscribed in arc CDB. Measure angle CDB.

Either through animation or by manually moving point D about the circle, observe the changes in your measure. When you feel you have enough information on which to form a hypothesis about the angle in the diagram, complete the remainder of this worksheet.

Be sure to save and/or print your sketch as directed by your teacher.

In Part I you investigated four types of angles: central, inscribed, inside, and outside. Which of these types of angles is angle CDB?

In what arc is angle CDB inscribed?

There are three types of arcs: minor arc, semicircle, major arc. Which of these types of arcs is arc CDB?

Complete the following:

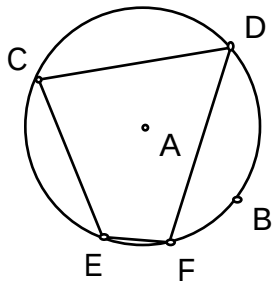
An angle inscribed in a _____ is _____.
(name the type of arc) (complete in your words)

In your own words, explain why this is true.

Angle-Arc Relationships in the Circle
Part II
Investigation #7

In the previous investigation you saw an angle inscribed in an arc. a **POLYGON** is **INSCRIBED IN A CIRCLE** when all vertices of the polygon are points of the circle.

Create a sketch like the one below. Be sure to save and/or print your sketch as directed by your teacher.



Measure angles DCE and EFD, then measure angles CEF and FDC. Using the calculator found under MEASURE, also find the sums of the first two and last two angles.

Either through animation or by manually moving points C, D, E, and/or F about the circle while maintaining the polygon shape, observe the changes in your measures. When you feel you have enough information on which to form a hypothesis about the angle sums in the diagram, complete the remainder of this worksheet.

Be sure to save and/or print your sketch.

How many sides does the polygon have? ____ What is the name of this type of polygon? _____

The pairs of angles whose sums you found were across from one another. What is the name for a pair of angles such as these? _____

Complete the following:

If a _____ is inscribed in a circle, the _____ angles
(name of polygon) (type of angles)

are _____.
(some angle relationship)

In your own words, explain why this is true. HINTS: What type of angles are these with respect to the circle? How is its measure found?